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Darkness to Light

TOPIC:- K-MAP AND Quine - McCluskey method of minimization.

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Karnaugh Maps

- Algebraic procedures:
    - Difficult to apply in a systematic way.
    - Difficult to tell when you have arrived at a minimum solution.
- Karnaugh map (K-map) can be used to minimize functions of up to 6 variables.
    - K-map is directly applied to two-level networks composed of AND and OR gates.
        - Sum-of-products, (SOP)
        - Product-of-sum, (POS).

Minimum SOP

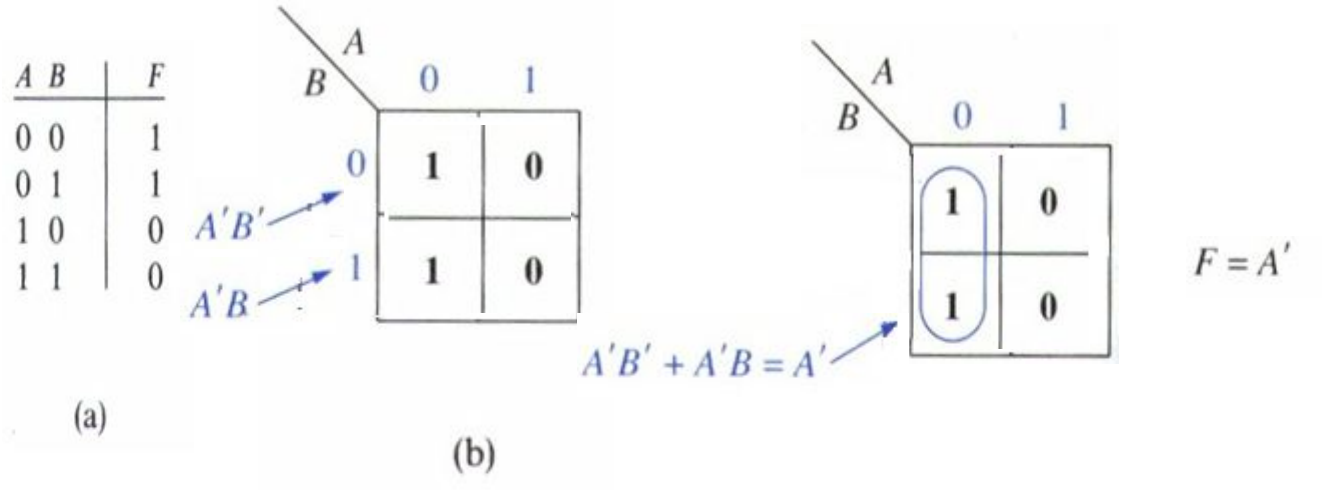
- It has a minimum no. of terms.
  - That is, it has a minimum number of gates.
- It has a minimum no. of gate inputs.
  - That is, minimum no. of literals.
  - Each term in the minimum SOP is a prime implicant, i.e., it cannot be combined with others.
- It may not be unique.
  - Depend on the order in which terms are combined or eliminated.

Minimum POS

- It has a minimum no. factors.
- It has a minimum no. of literals.
- It may not be unique.
  - Use (X+Y) (X+Y’) = X

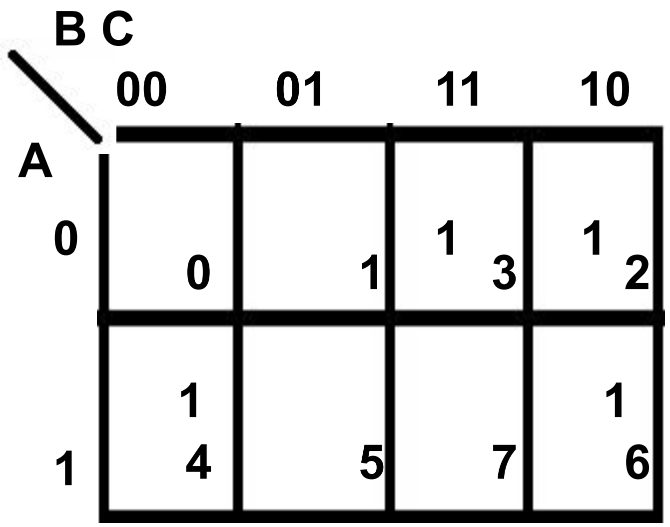
2-Variable K-map

- Place 1s and 0s from the truth table in the K-map.
- Each square of 1s = minterms.
- Minterms in adjacent squares can be combined since they differ in only one variable.



3-Variable K-map

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

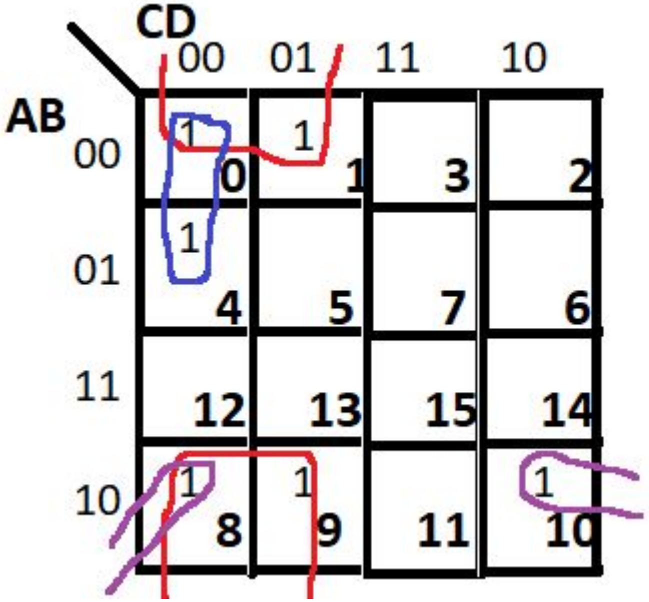


$$\bar{A}B + B\bar{C} + A\bar{C}$$

4-Variable K Map

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

$$f(A,B,C,D) = \sum m(0,1,4,8,9,10)$$



$$\bar{B}\,\bar{C} + \bar{A}\,\bar{C}\,\bar{D} + A\,\bar{B}\,\bar{D}$$

Q. Minimize using k-map:-

$$\bar{A}\bar{B}C\bar{D} + ABC\bar{D} + A\bar{B}C\bar{D} + A\bar{B}CD + A\bar{B}\bar{C}\bar{D} + AB\bar{C}\bar{D} + \bar{A}\bar{B}CD + \bar{A}\bar{B}\bar{C}\bar{D}$$

$\bar{A}\bar{B}C\bar{D}$	$ABC\bar{D}$	$A\bar{B}C\bar{D}$	$A\bar{B}CD$	$A\bar{B}\bar{C}\bar{D}$	$AB\bar{C}\bar{D}$	$\bar{A}\bar{B}CD$	$\bar{A}\bar{B}\bar{C}\bar{D}$
0010	1110	1010	1011	1000	1100	0011	0000
2	14	10	11	8	12	3	0

ROUGH

$\bar{D} = 0$

$D = 1$

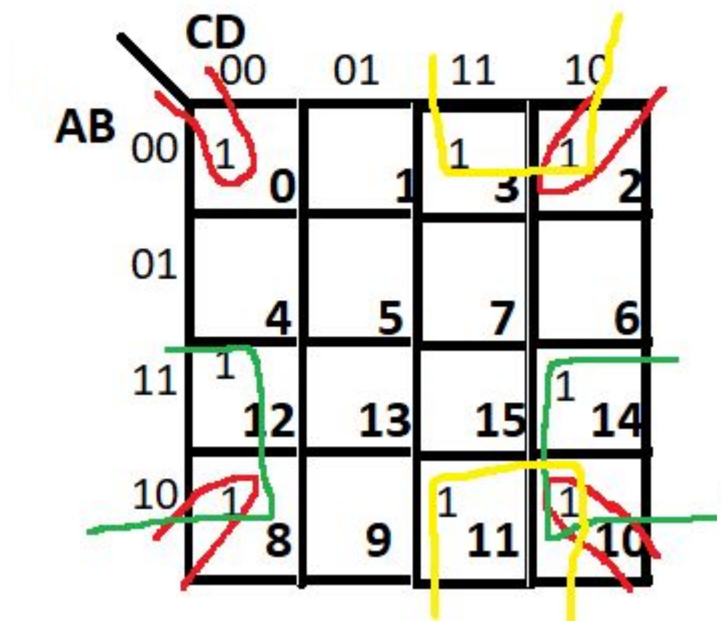
$\bar{A}\ \bar{B}\ C\ \bar{D}$

0 0 1 0

8 4 2 1

1 1 1 0

8 + 4 +2 = 14



$\bar{B}\bar{D} + A\bar{D} + \bar{B}C$

Quine-McCluskey Method

- The Quine–McCluskey method is a systematic algorithm, also known as the Tabular method, for simplifying Boolean expressions with many variables, making it more suitable for machine computation than Karnaugh maps (K-maps) which are limited to a smaller number of inputs.
- A prime implicant chart is used to select a minimum set of prime implicants.

$$f(A,B,C,D) = \Sigma(1,2,5,6,7,9,10) + \Sigma d(0,13,15)$$

Implication Table

Column I

0	0000
1	0001
2	0010
5	0101
6	0110
9	1001
10	1010
7	0111
13	1101
15	1111

$$f(A,B,C,D) = \Sigma(1,2.5,6,7,9,10) + \Sigma d(0,13,15)$$

Implication Table

Column I		Column II	
0	0000	000-	(0,1)
1	0001	00-0	(0,2)
2	0010	0-01	(1,5)
5	0101	-001	(1,9)
6	0110	0-10	(2,6)
9	1001	-010	(2,10)
10	1010	01-1	(5,7)
7	0111	-101	(5,13)
13	1101	011-	(6,7)
15	1111	1-01	(9,13)
		-111	(7,15)
		11-1	(13,15)

$f(A,B,C,D) = \Sigma(1,2.5,6,7,9,10) + \Sigma d(0,13,15)$

Implication Table

Column I		Column II		Column III	
0	0000	000-	(0,1)	--01	(1,5,9,13)
1	0001	00-0	(0,2)	-1-1	(5,7,13,15)
2	0010	0-01	(1,5)	00-0	(0,2)
5	0101	-001	(1,9)		
6	0110	0-10	(2,6)	0-10	(2,6)
9	1001	-010	(2,10)	-010	(2,10)
10	1010	01-1	(5,7)	011-	(6,7)
7	0111	-101	(5,13)	000-	(0,1)
13	1101	011-	(6,7)		
15	1111	1-01	(9,13)		
		-111	(7,15)		
		11-1	(13,15)		

Prime Implicant

Prime Implicant chart

	1	2	5	6	7	9	10
(1,5,9,13)	X		X			X	
(5,7,13,15)							
(0,2)							
(2,6)							
(2,10)							
(6,7)							
(0,1)							

Essential prime implicant

--01 $\bar{C}D$

-010 $\bar{B}C\bar{D}$

→ 011- $\bar{A}BC$

$\bar{A}BC + \bar{B}C\bar{D} + \bar{C}D$