DEPARTMENT - ECE
SUBJECT- Network Theory

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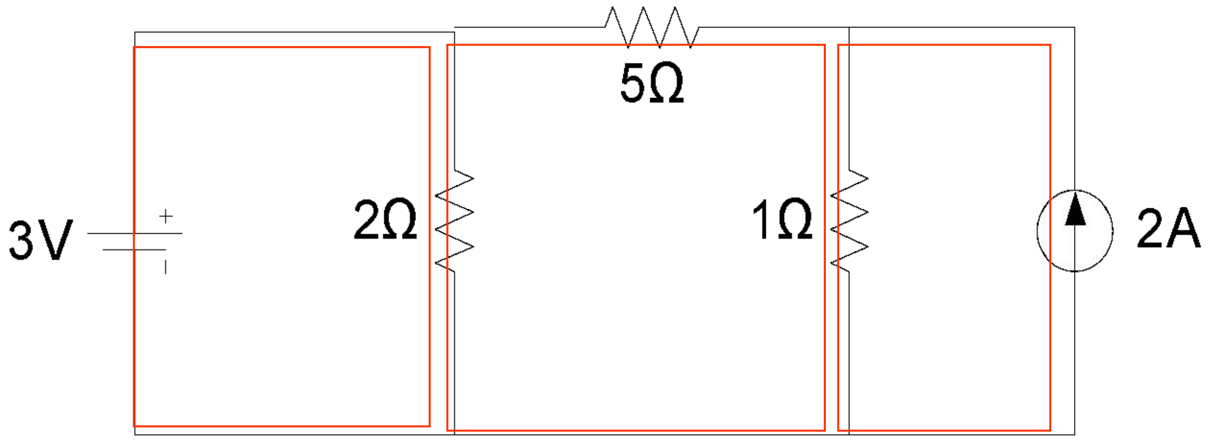
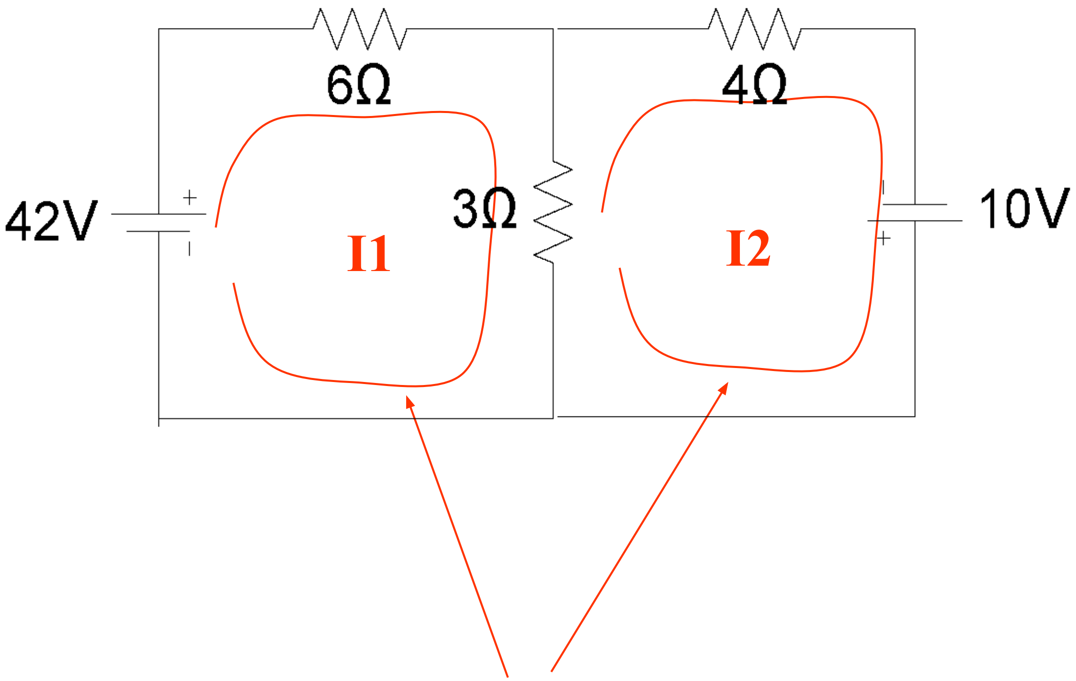
Topic:-Mesh Analysis (Loop Analysis)

Mesh = A closed loop path which has no smaller loops inside



Mesh currents are circular currents used for calculation.



Mesh current (loop current)

Real current is a sum of all mesh currents passing through that point.

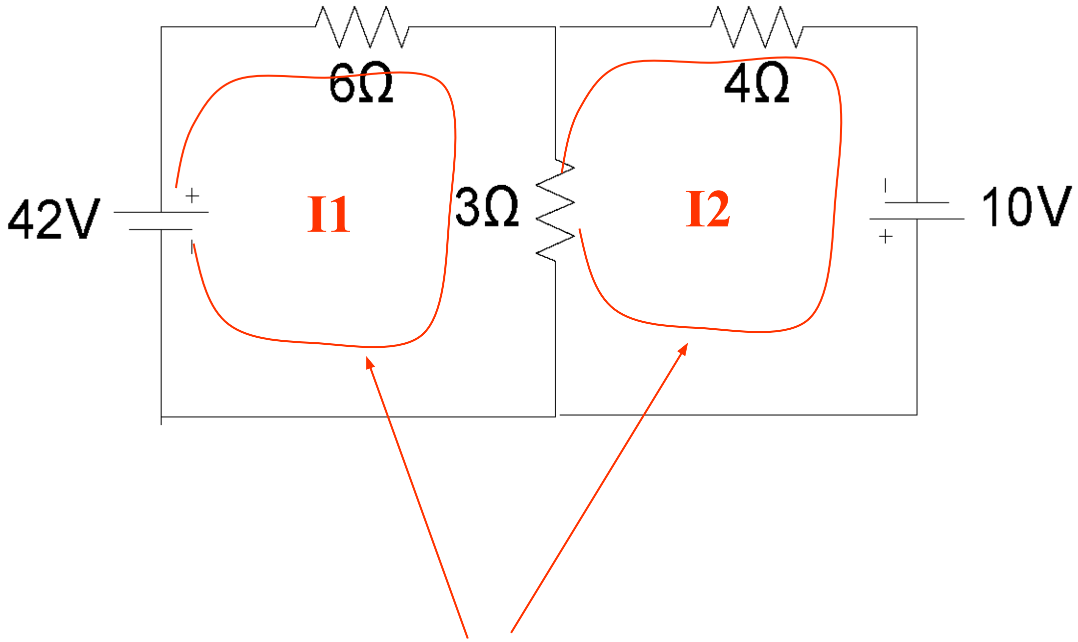
Mesh Analysis

<u>Procedure</u>

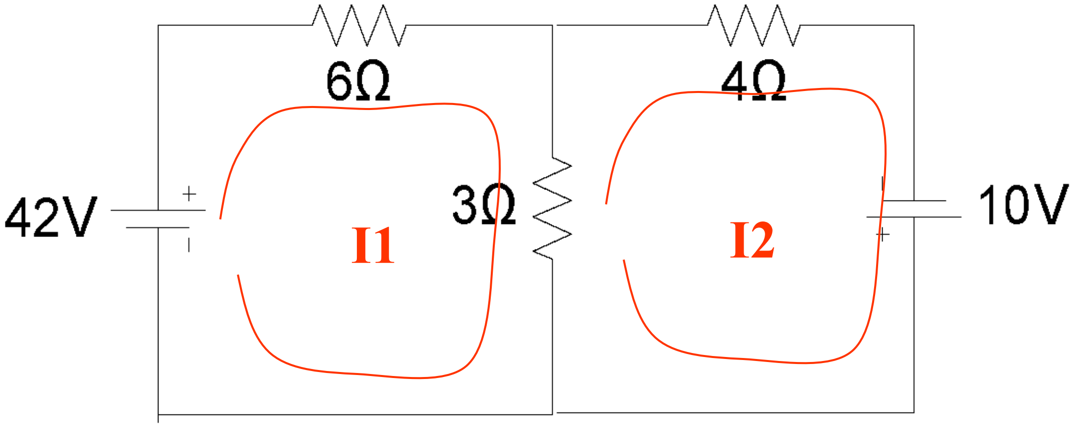
1. **Count the number of meshes. Let the number equal N.**
2. **Define mesh current on each mesh. Let the values be I1, I2, I3, …**
3. **Use Kirchoff’s voltage law (KVL) on each mesh, generating N equations**
4. **Solve the equations**

Example

Use mesh analysis to find the power consumption in the resistor 3 Ω



Mesh current (loop current)



Loop 1

$$-42 + 6I1 + 3(I1 - I2) = 0$$

$$9I1 - 3I2 = 42$$

Equation 1

Loop 2

$$3(I2 - I1) + 4I2 - 10 = 0$$

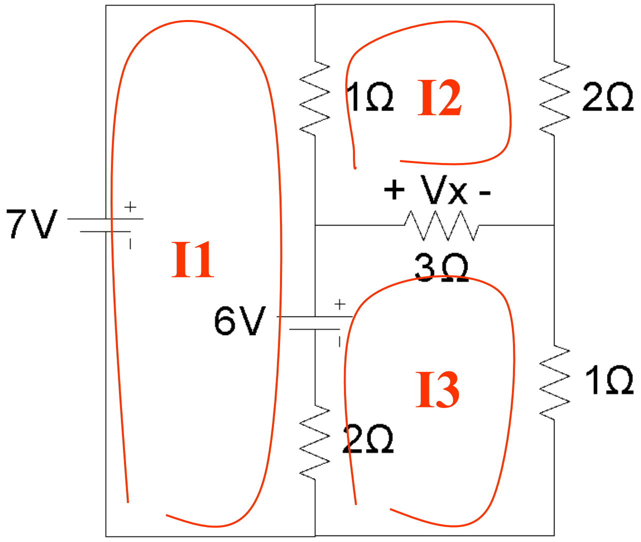
$$-3I1 + 7I2 = 10$$

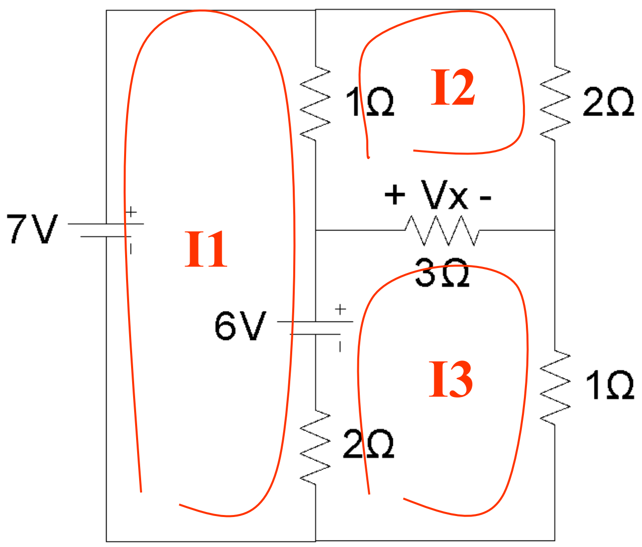
Equation 2

I1 = 6A, I2 = 4A, The current that pass through R 3Ω is 6-4 = 2A (downw [redacted]
Power = 12 W

Example

Use Mesh analysis to find Vx





$$-7+1(I1-I2)+6+2(I1-I3)=0$$
$$3I1-I2-2I3=1$$

Equation 1

$$1(I2-I1)+2I2+3(I2-I3)=0$$
$$-I1+6I2-3I3=0$$

Equation 2

$$2(I3-I1)-6+3(I3-I2)+I3=0$$
$$-2I1-3I2+6I3=6$$

Equation 3

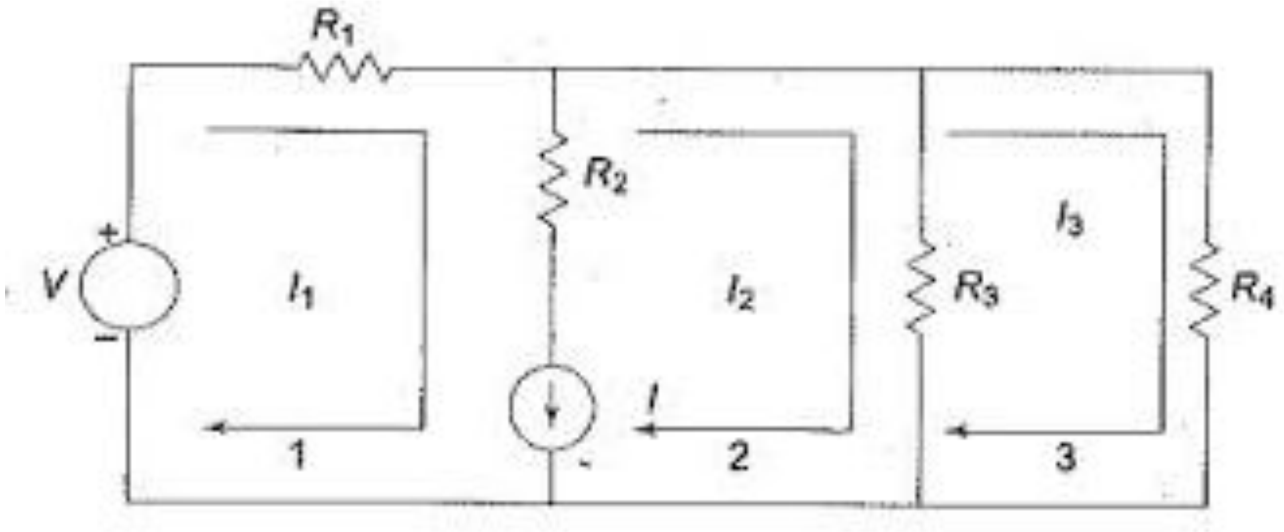
I1 = 3A, I2 = 2A, I3 = 3A

Vx = 3(I3-I2) = 3V

Supermesh

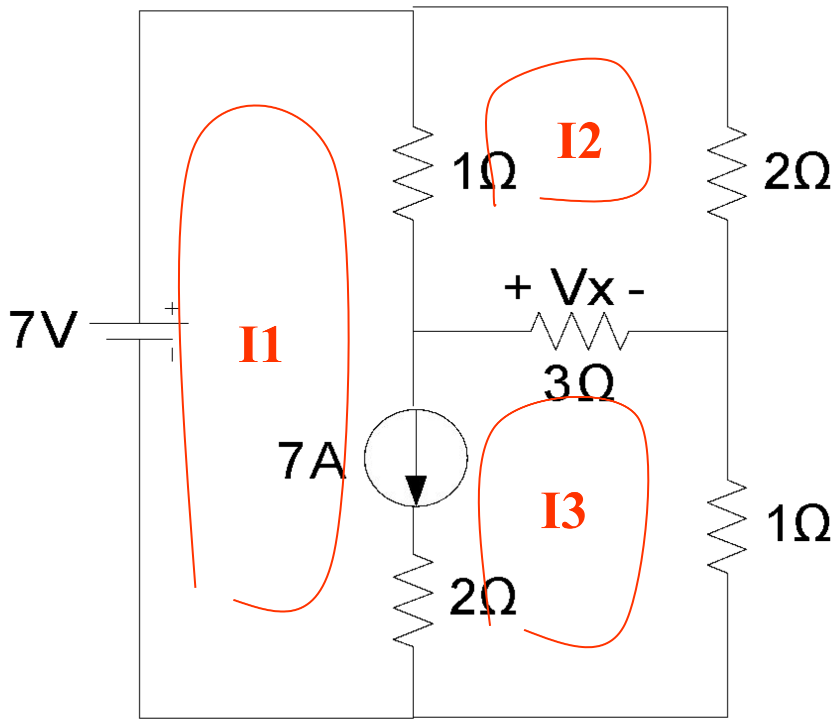
When there is a current source in the mesh path, we cannot use KVL because we do not know the voltage across the current source.

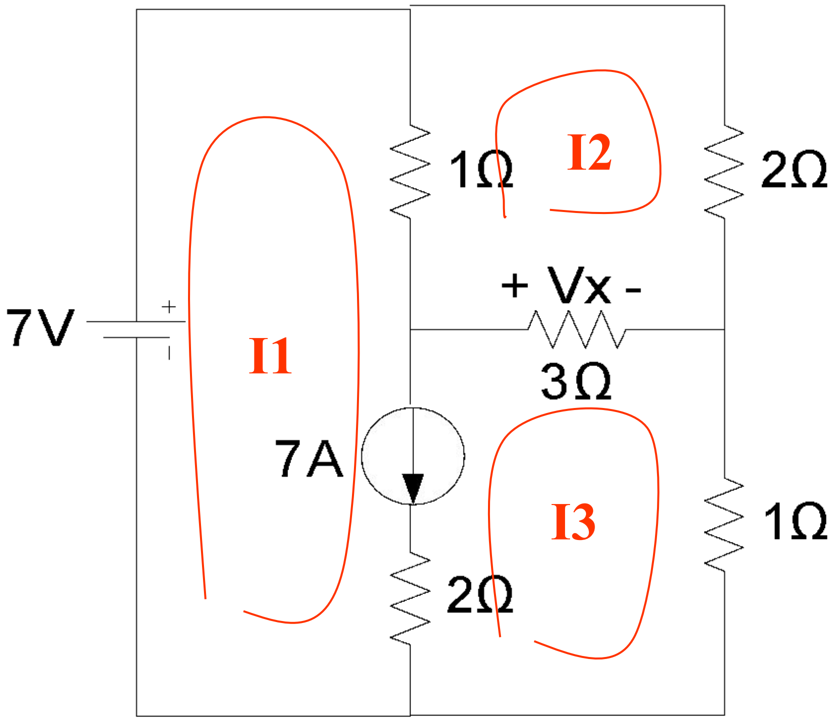
We have to use supermesh, which is a combination of 2 meshes to be a big mesh, and avoid the inclusion of the current source in the mesh path.



Example

Use Mesh analysis to find Vx

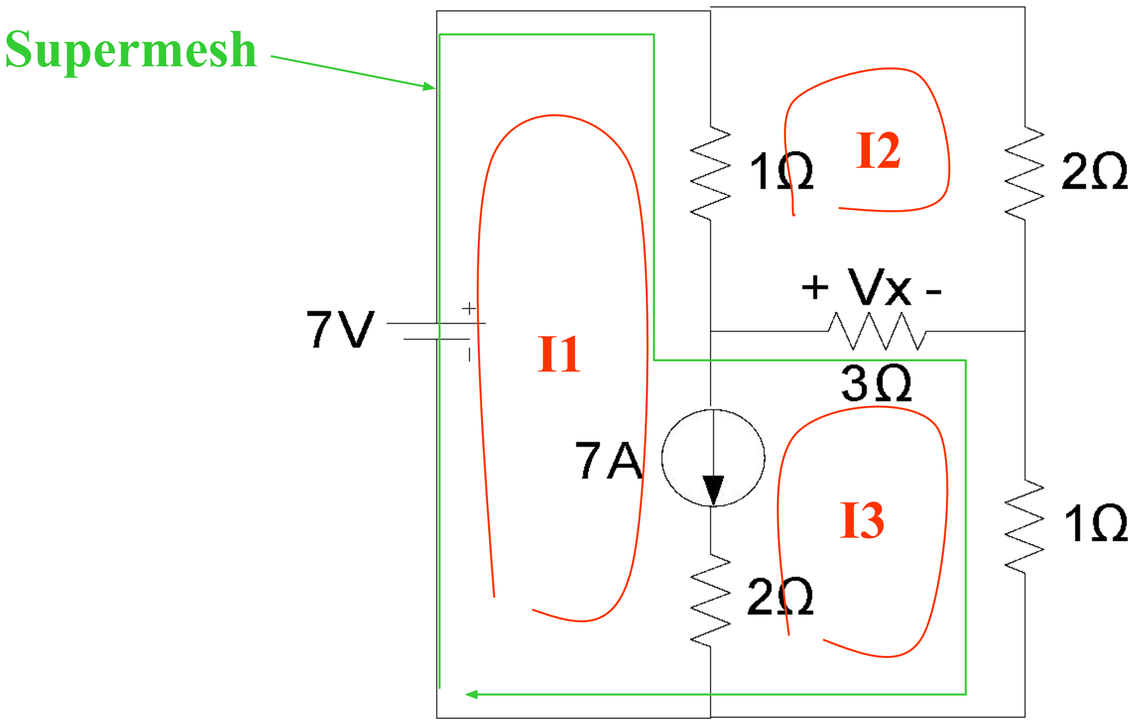




$$1(I2 - I1) + 2I2 + 3(I2 - I3) = 0$$

$$-I1 + 6I2 - 3I3 = 0$$

Equation from 2nd [redacted]



$$-7+1(I1-I2)+3(I3-I2)+I3=0$$

$$I1-4I2+4I3=7$$ **Equation 2**

$$I1-I3=7$$ **Equation 3**

$I1 = 9A$

$I2 = 2.5A$

$I3 = 2A$

$Vx = 3(I3-I2) = -1.5V$

Nodal Voltage Method

Nodal voltage analysis applies **KCL** to find unknown voltages.

A node is the point of connection between two or more branches.

The nodal voltage method is applied as follows:

Step 1 : Convert each voltage source in the network to its equivalent current source.

Step 2 : Determine the number of nodes within the network.

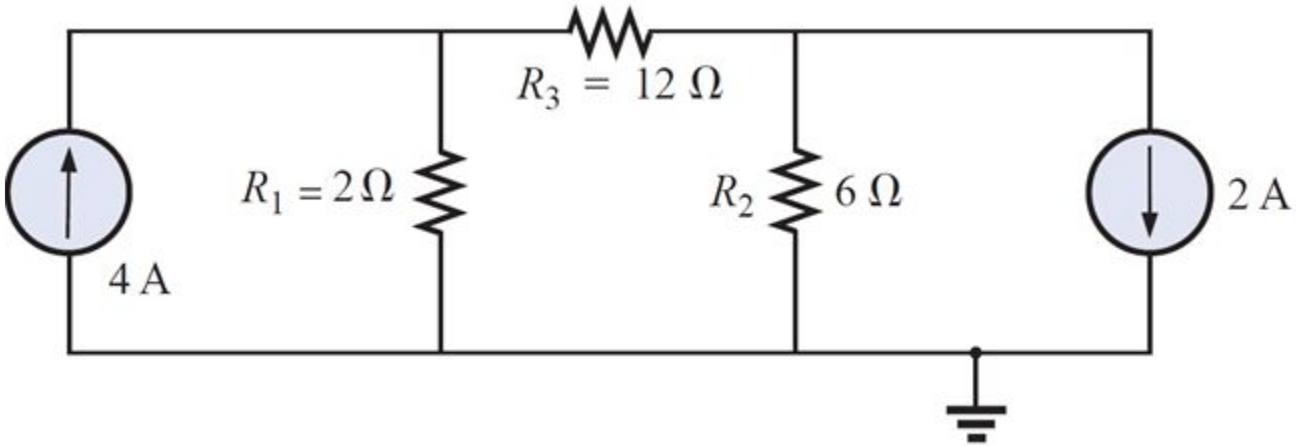
Step 3 : Pick a reference node, and label each remaining node with a subscripted value of voltage: V1, V2, and so on.

Step 4 : Apply Kirchhoff's current law at each node except the reference.

Step 5 : Solve the resulting equations to obtain the unknown node voltages.

Nodal Voltage Method

EXAMPLE: Using the nodal voltage method, determine the currents in $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, in the circuit shown.

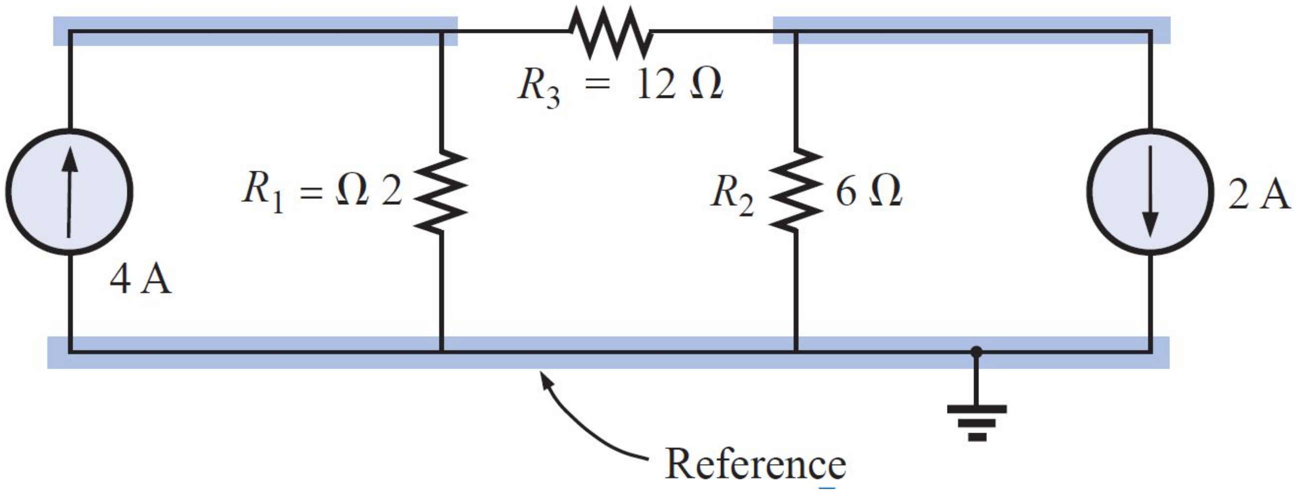


Nodal Voltage Method

EXAMPLE: Using the nodal voltage method, determine the currents in $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, in the circuit shown.

Solution:

Step 2 : Determine the number of nodes within the network.

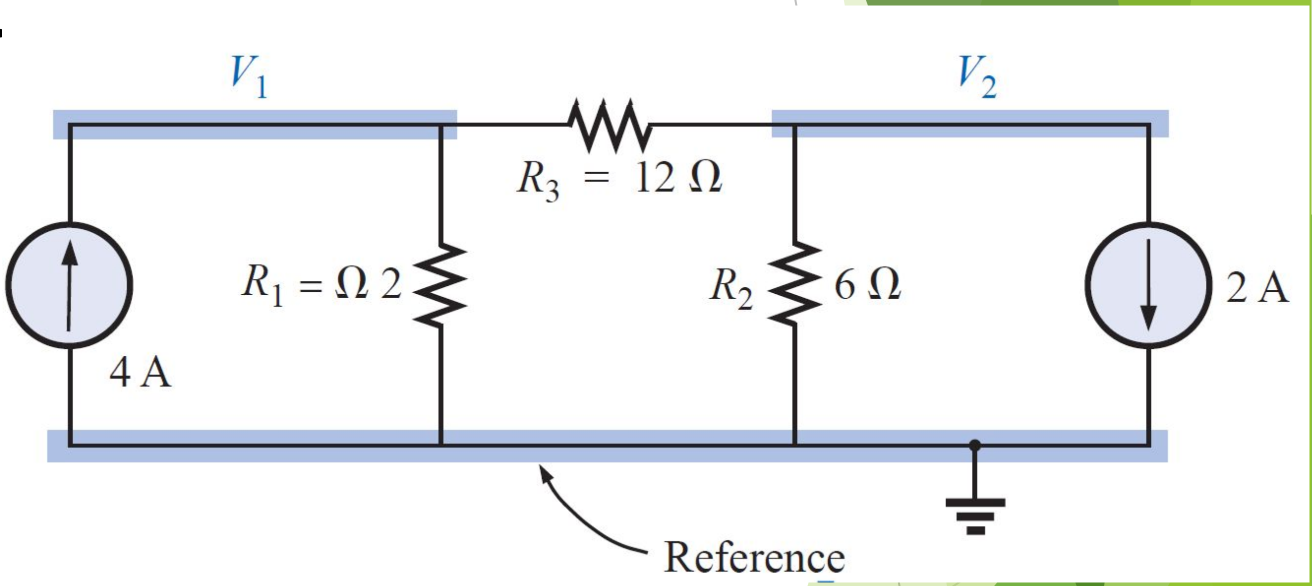


Nodal Voltage Method

EXAMPLE: Using the nodal voltage method, determine the currents in $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, in the circuit shown.

Solution:

Step 3 : **Pick a reference node, and label each remaining node with a subscripted value of voltage: V1, V2, and so on.**

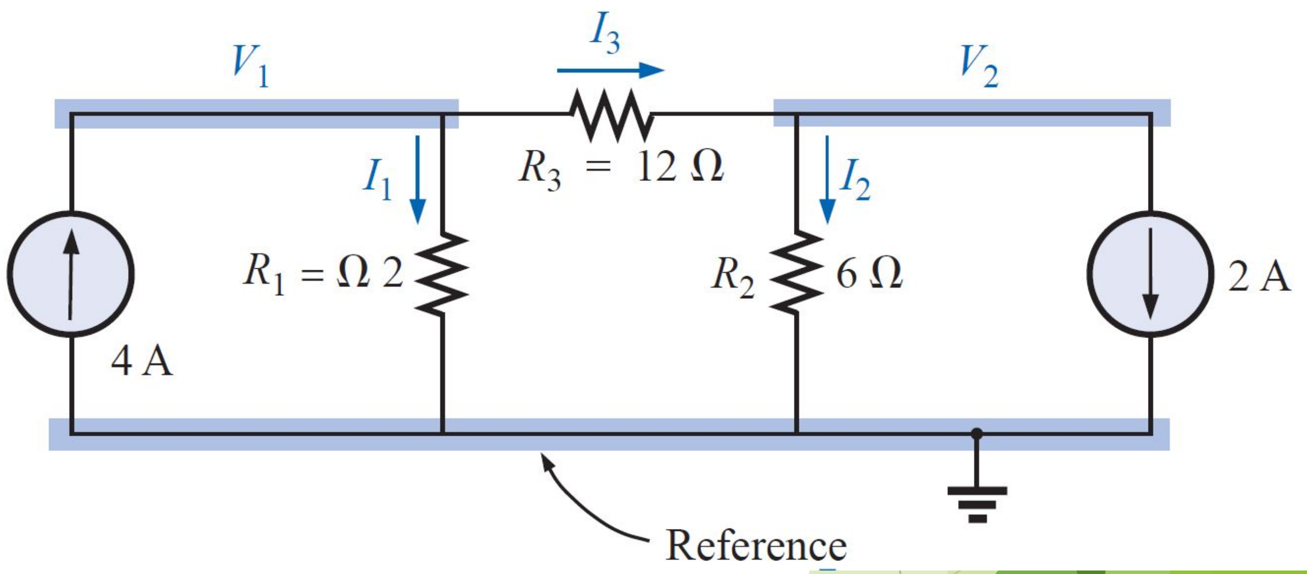


Nodal Voltage Method

EXAMPLE: Using the nodal voltage method, determine the currents in $\mathbf{R_1}$, $\mathbf{R_2}$ and $\mathbf{R_3}$, in the circuit shown.

Solution:

Step 4 : Apply Kirchhoff's current law at each node except the reference.



For node 1 :

Applying Kirchhoff's current law:

$$4\text{ A} - I_1 - I_3 = 0$$

$$4\text{ A} = I_1 + I_3$$

and
$$4\text{ A} = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_3}$$

$$= \frac{V_1}{2\ \Omega} + \frac{V_1 - V_2}{12\ \Omega}$$

Expanding and rearranging:

$$V_1\left(\frac{1}{2\ \Omega} + \frac{1}{12\ \Omega}\right) - V_2\left(\frac{1}{12\ \Omega}\right) = 4\text{ A}$$

FOR NODE 2:- Applying Kirchhoff's current law:

$$I_3 - I_2 - 2\,\text{A} = 0$$

$$-2\,\text{A} = -I_3 + I_2$$

and $$-2\,\text{A} = \frac{V_2 - V_1}{R_3} + \frac{V_2}{R_2}$$

$$= \frac{V_2 - V_1}{12\,\Omega} + \frac{V_2}{6\,\Omega}$$

Expanding and rearranging:

$$V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) = -2\,\text{A}$$

NOW Solve the resulting equations to obtain the unknown node voltages:-

$$\left.\begin{aligned} V_1\left(\frac{1}{2\,\Omega} + \frac{1}{12\,\Omega}\right) - V_2\left(\frac{1}{12\,\Omega}\right) &= +4\,\text{A} \\ V_2\left(\frac{1}{12\,\Omega} + \frac{1}{6\,\Omega}\right) - V_1\left(\frac{1}{12\,\Omega}\right) &= -2\,\text{A} \end{aligned}\right\}$$

producing

$$\left.\begin{aligned} \frac{7}{12}V_1 - \frac{1}{12}V_2 &= +4 \\ -\frac{1}{12}V_1 + \frac{3}{12}V_2 &= -2 \end{aligned}\right\} \begin{aligned} 7V_1 - V_2 &= 48 \\ -1V_1 + 3V_2 &= -24 \end{aligned}$$

Solve the resulting equations to obtain the unknown node voltages.

$$7V_1 - V_2 = 48$$
$$-1V_1 + 3V_2 = -24$$

$$\Delta = \begin{vmatrix} +7 & -1 \\ -1 & +3 \end{vmatrix} = (7)(3) - (-1)(-1) = 21 - 1 = +20$$

$$\Delta 1 = \begin{vmatrix} +48 & -1 \\ -24 & +3 \end{vmatrix} = (48)(3) - (-1)(-24) = 144 - 24 = 120$$

$$\Delta 2 = \begin{vmatrix} +7 & +48 \\ -1 & -24 \end{vmatrix} = (7)(-24) - (48)(-1) = -168 + 48 = -120$$

$$\therefore V_1 = \frac{\Delta 1}{\Delta} = \frac{120}{+20} = +6\ \mathbf{V}$$

$$\text{and} \quad V_2 = \frac{\Delta 2}{\Delta} = \frac{-120}{+20} = -6\ \boldsymbol{V}$$

Now determine the value of currents:-

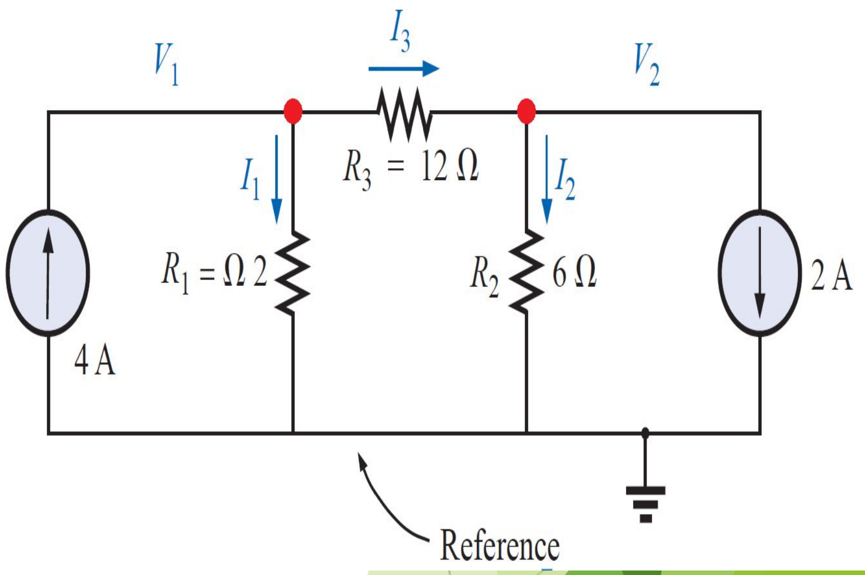
$$I_{R1} = I_1 = \frac{V_1}{R_1} = \frac{+6}{+2} = \mathbf{+3\ A}$$

$$I_{R2} = I_2 = \frac{V_2}{R_2} = \frac{-6}{+6} = \mathbf{-1\ A}$$

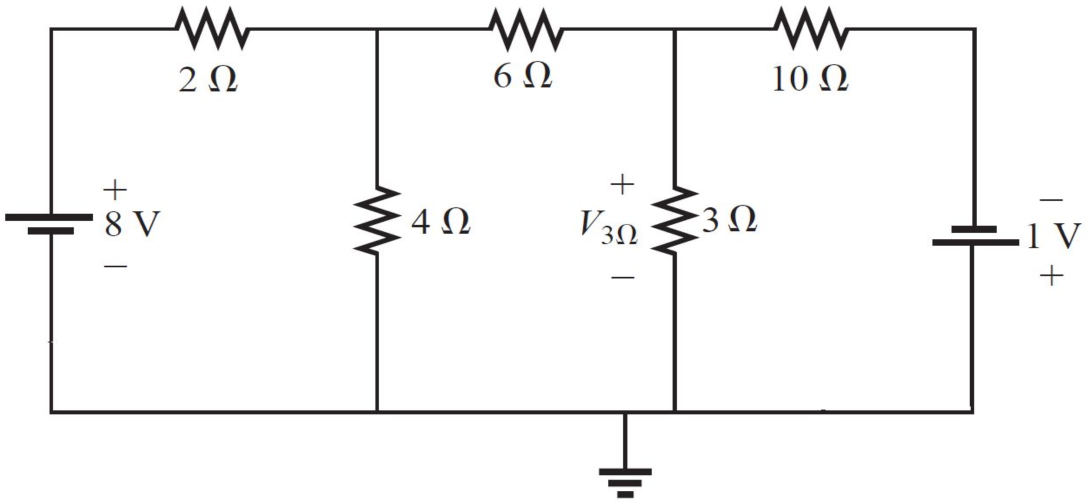
The **minus signs** indicate that the current has opposite direction

Since V_1 is greater than V_2, the current through R_3 passes from V_1 to V_2. Its value is

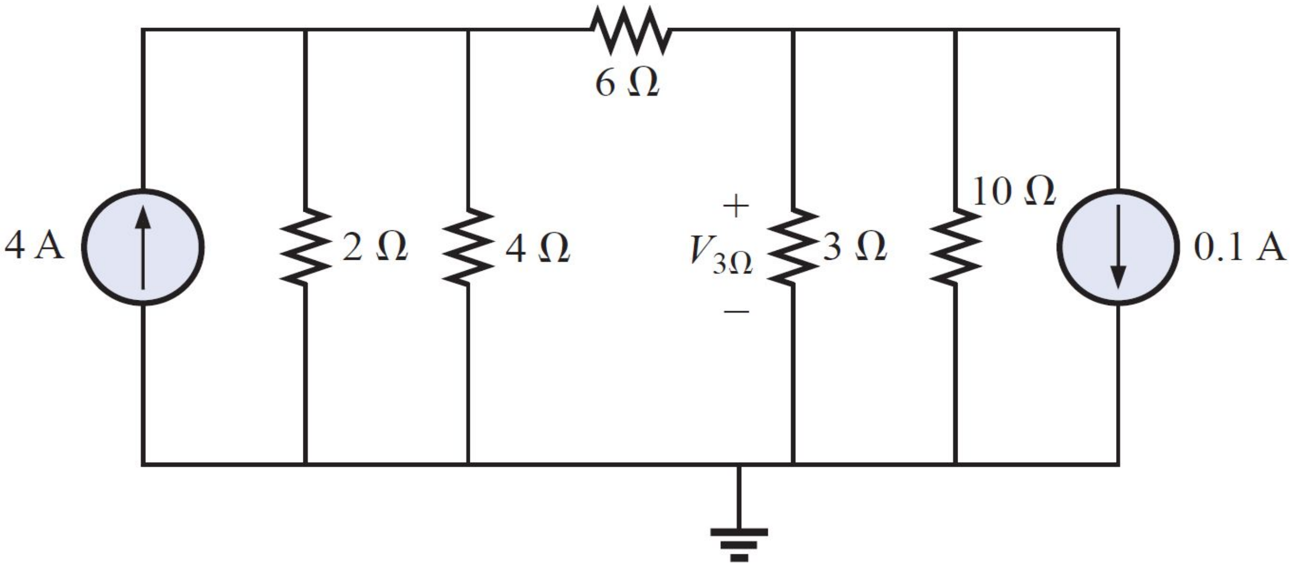
$$I_{R3} = I_3 = \frac{V_1 - V_2}{R_3} = \frac{+6-(-6)}{+12} = \frac{+12}{+12} = \mathbf{+1\ A}$$

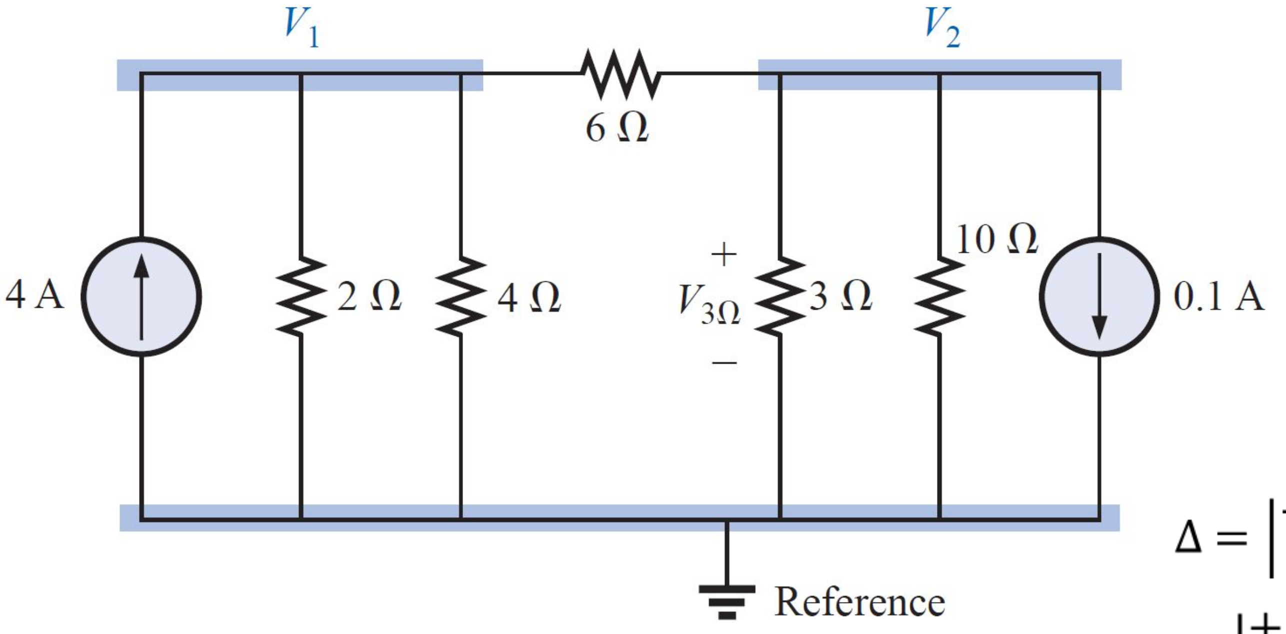


EXAMPLE: By using nodal voltage method, find the voltage across the **3 Ω** resistor of the circuit shown.



Converting sources and choosing nodes, we have





$$\left(\frac{1}{2\,\Omega}+\frac{1}{4\,\Omega}+\frac{1}{6\,\Omega}\right)V_1-\left(\frac{1}{6\,\Omega}\right)V_2=+4\text{ A}$$
$$\left(\frac{1}{10\,\Omega}+\frac{1}{3\,\Omega}+\frac{1}{6\,\Omega}\right)V_2-\left(\frac{1}{6\,\Omega}\right)V_1=-0.1\text{ A}$$

$$\frac{11}{12}V_1-\frac{1}{6}V_2=4$$
$$-\frac{1}{6}V_1+\frac{3}{5}V_2=-0.1$$

Resulting in

$$11V_1 - 2V_2 = +48$$
$$-5V_1 + 18V_2 = -3$$

Solve the resulting equations to obtain the unknown node voltages.

$$\Delta=\begin{vmatrix}+11 & -2\\ -5 & +18\end{vmatrix}=(11)(18)-(-2)(-5)=198-10=+188$$

$$\Delta 2=\begin{vmatrix}+11 & +48\\ -5 & -3\end{vmatrix}=(11)(-3)-(48)(-5)=-33+240=+207$$

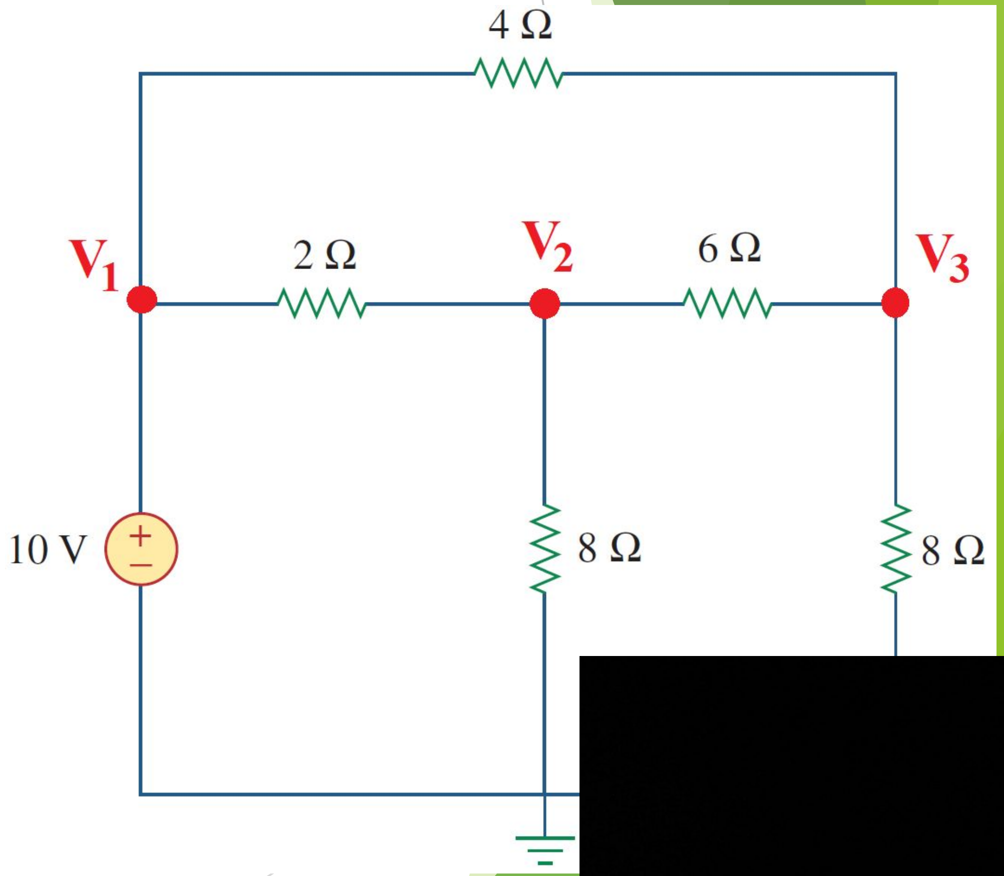
$$V_{3\Omega}=V_2=\frac{\Delta 2}{\Delta}=\frac{+207}{+188}=-1.101\ \boldsymbol{V}$$

Special cases

CASE 1 **If a voltage source is connected between the reference node and a nonreference node, we simply set the voltage at the nonreference node equal to the voltage of the voltage source.**

In this circuit, we have 3 node voltages: $V_1, V_2, and\ V_3$

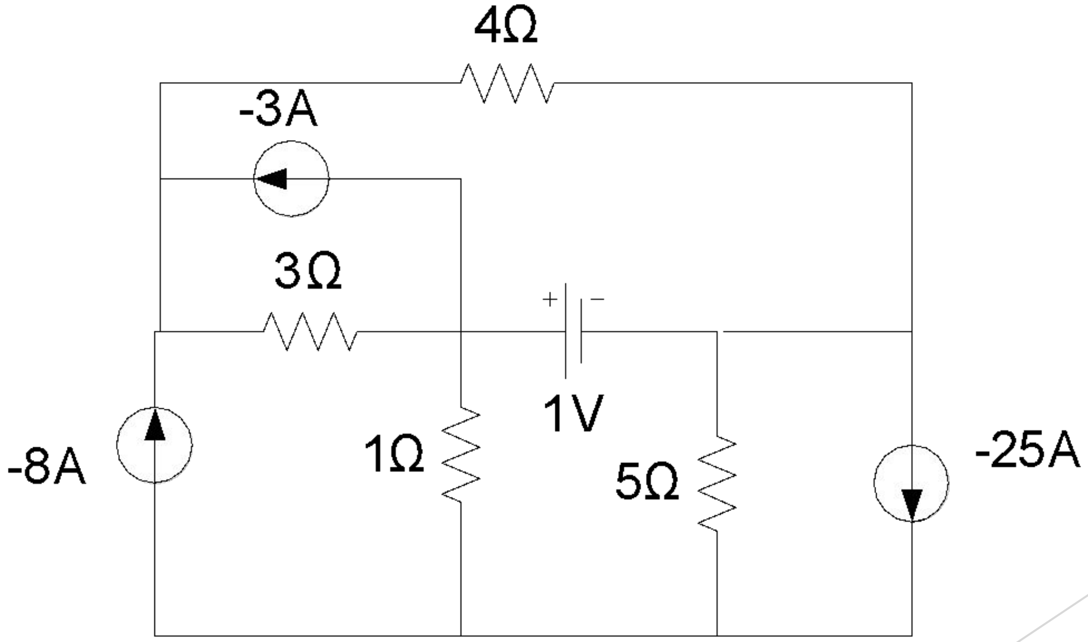
$Here \quad V_1 = 10\ V$

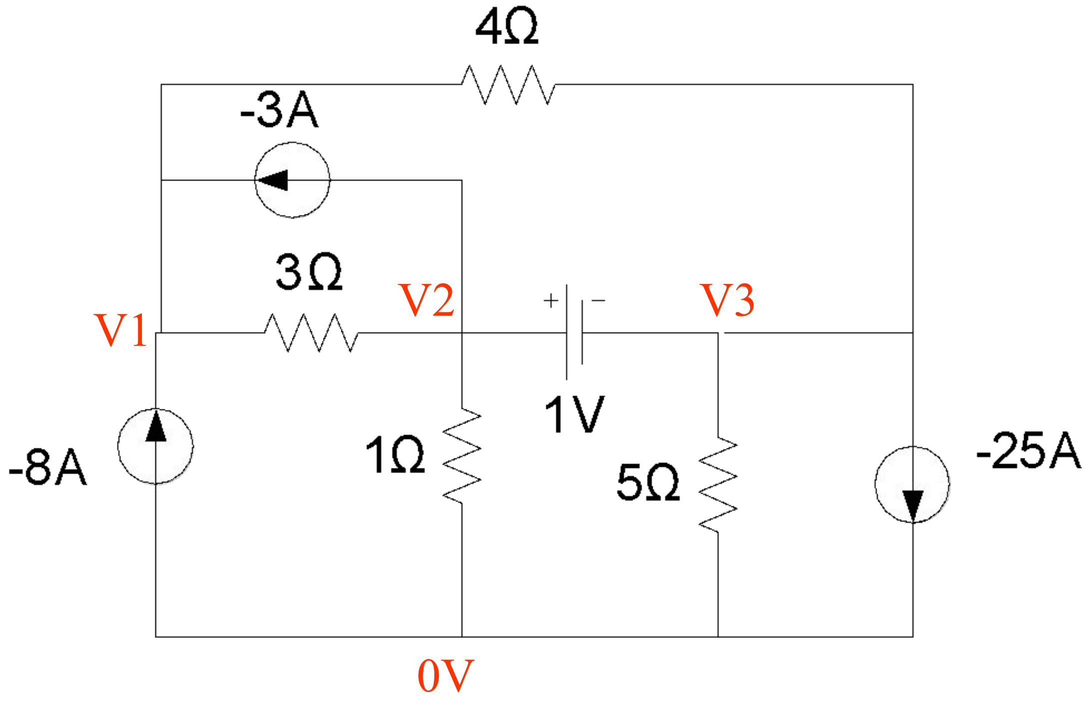


Supernode

When there is a voltage source in the circuit, direct KCL cannot be used because we do not know the current in the voltage source.Than,We will use the idea of supernode.

Supernode is the method that combines 2 nodes together when using KCL. It will include the voltage source within the circle when using KCL.

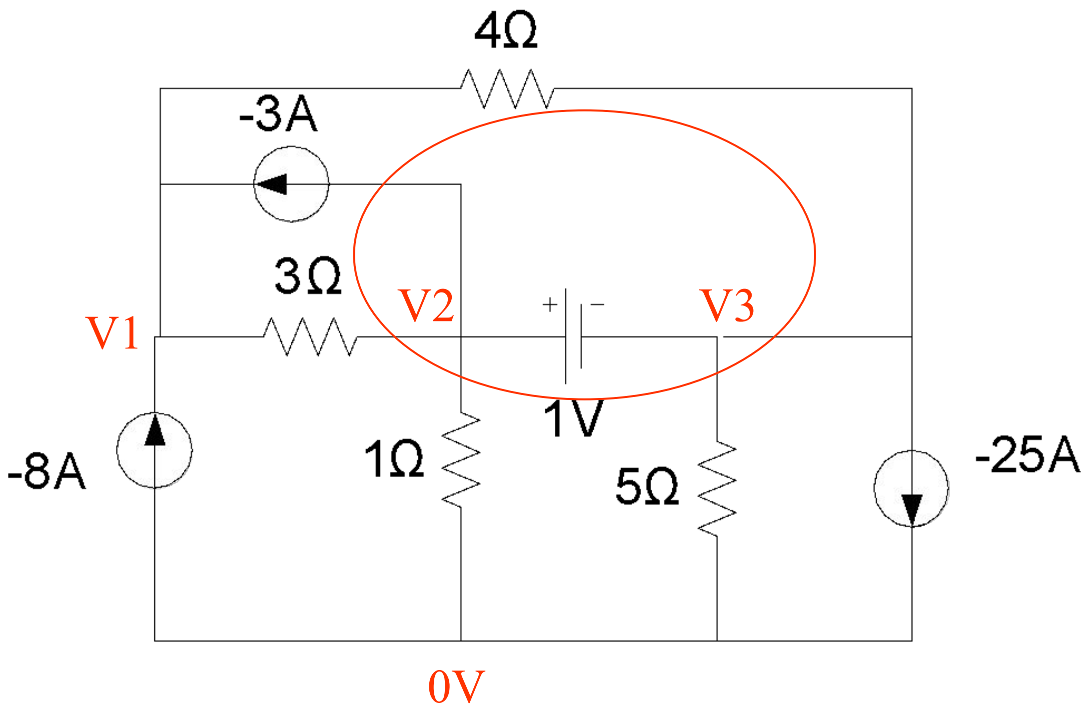




$$8+\frac{V1-V3}{4}+3+\frac{V1-V2}{3}=0$$

$$96+3V1-3V3+36+4V1-4V2=0$$

$$7V1-4V2-3V3=-132$$ **Equation 1**



$$\frac{V2-V1}{3}-3+\frac{V3-V1}{4}-25+\frac{V3-0}{5}+\frac{V2-0}{1}=0$$

$$20V2-20V1-180+15V3-15V1-1500+12V3+60V2=0$$

$$-35V1+80V2+27V3=1680$$ **Equation 2**

$$V2-V3=1$$ **Equation 3**

By Solving These three equation :-

$$7V1 - 4V2 - 3V3 = -132$$

$$-35V1 + 80V2 + 27V3 = 1680$$

$$V2 - V3 = 1$$

We will find out:-

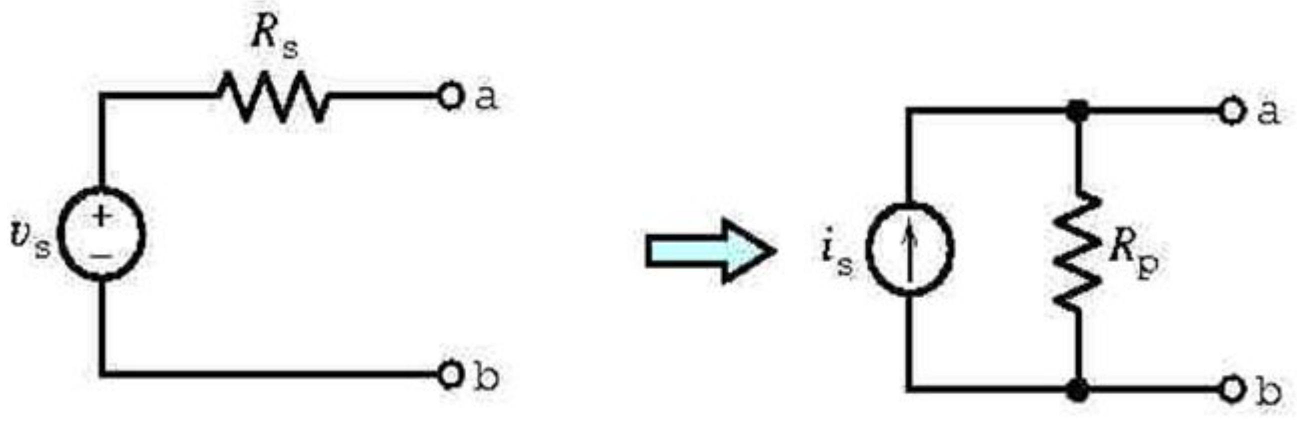
V1 = -4.952 V

V2 = 14.333 V

V3 = 13.333 V

Source Transformations

A *source transformation* is a procedure for transforming one source into another while retaining the terminal characteristics of the original source.



Set $i_s = \dfrac{v_s}{R_s}$

Set $R_p = R_s$

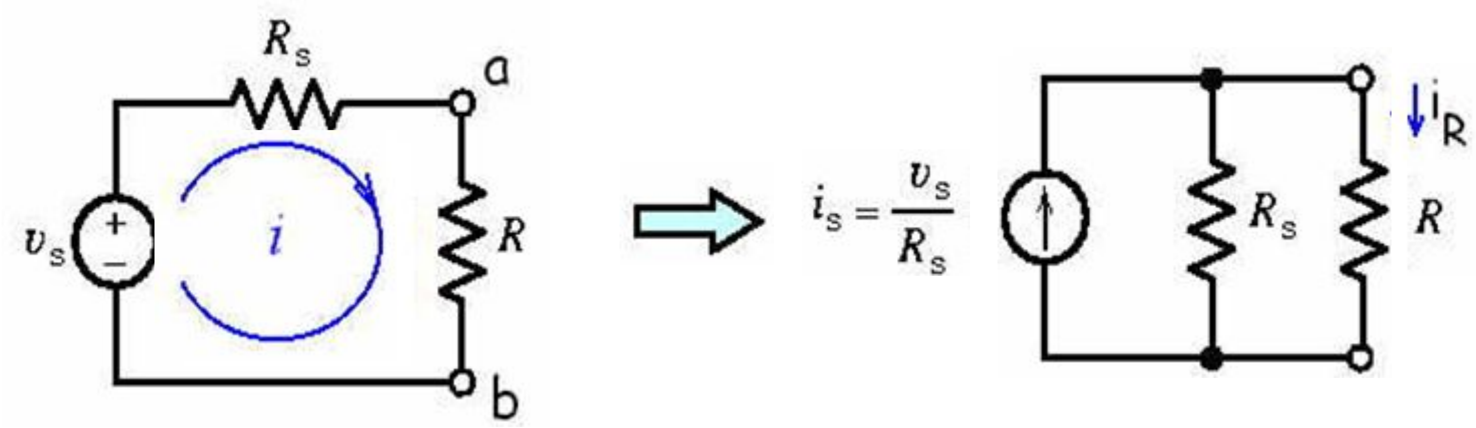
Duality and Dual Circuits

Circuits are said to be ***dual*** when the characterizing equations of one network can be obtained from the other by simply interchanging *v* and *i* and interchanging *G* and *R*.

Duality Pairs:

Resistance	⟺	Conductance
Current	⟺	Voltage
Series	⟺	Parallel
Etc.		

The equivalence of the transformed circuits can be verified by showing that the current flowing in a load resistor connected between terminals a-b is identical for both circuits.



Mesh analysis:

$$-V_s + R_s i + Ri = 0$$

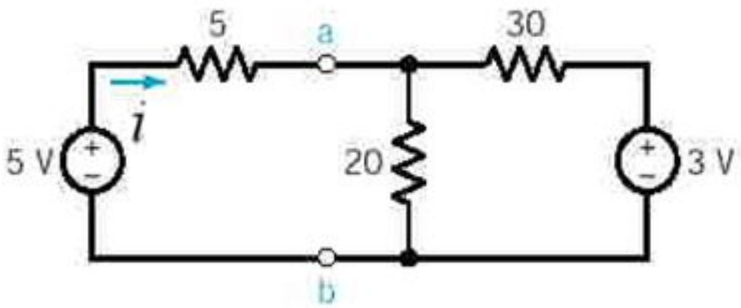
or $i = \frac{V_s}{R_s + R}$

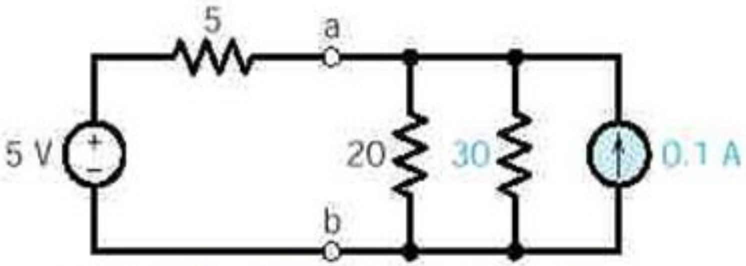
By current division:

$$i_R = \frac{R_s i_s}{R_s + R} = \frac{R_s}{R_s + R}\frac{V_s}{R_s}$$

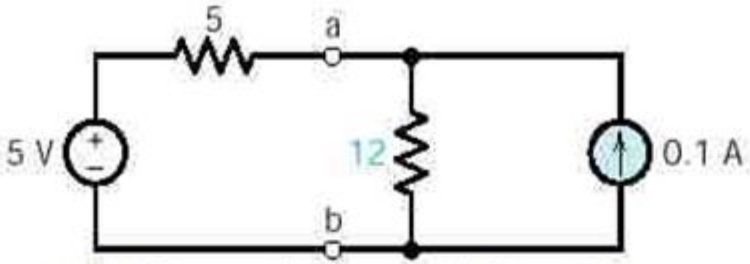
$$i_R = \frac{V_s}{R_s + R}$$

Example: Find the current i by reducing the circuit to the right of terminals a-b to its simplest form using source transformations.

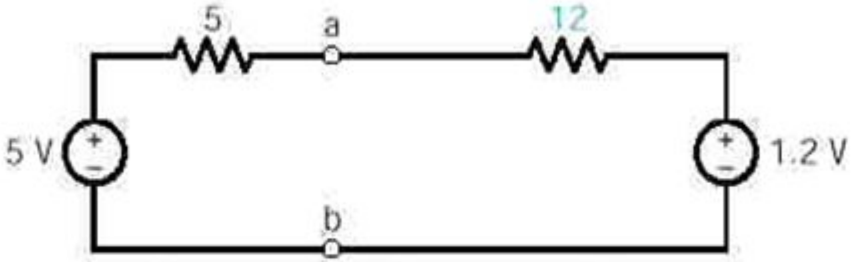




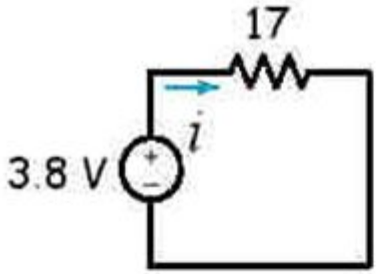
step #1: transform 3V source



step#2: combine parallel resistors



step #3: circuit after transforming the 0.1 A source



step #4: circuit after combining the two sources and the series resistors.

Finally, by Ohm's law:

$$i = 3.8/17 = 0.224 \text{ A}$$